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Deployment Considerations for Spacecraft Formation at Sun-Earth L2 Point

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Abstract: The coordination and control of a constellation of spacecraft, flying a few meters from one another, dictates several interesting design requirements, including efficient architectures and algorithms for formation acquisition, reorientation and resizing. The spacecraft must perform these transitions without interfering or colliding into each other. Furthermore position keeping is fundamental for formation efficiency. This paper presents an optimal deployment of the DARWIN formation using the potential function control technique in the vicinity of the Sun-Earth L2 point. The method hinges on defining a potential function from the geometric configuration of the constellation together with any collision avoidance requirement. A review of the fundamentals of relative motion and dynamics is presented before describing the features of the different control algorithms and validating the method using Lyapunov's theorem. The potential function method has been used to control both translational and rotational control. Obstacles, in the shape of other satellites and constrained payload pointing directions have been included. Finally it will be shown that the attitude control algorithm can be successfully used to avoid plume impingement that can have catastrophic consequences for the mission.

Keywords: *Formation flying; Lyapunov functions; rotational control.*

Mathematics Subject Classification (2000): 70E55, 70F15, 70M20.

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1 Introduction

Over the past decade, the introduction of cost reduction policy by the major space agencies caused a paradigm shift in the design of scientific satellites, as the primary metric by which spacecraft were judged switched from purely performance to specific performance or performance per unit cost [Cyrus and Miller, 1997]. Several new technologies, including multifunctional structures, micro-electro-mechanical systems, nano-technology and distributed satellite systems, have the potential to revolutionise the field of satellite design. In particular in the field of distributed satellite systems, that is systems based on dividing the tasks among several light and small satellites, two approaches exist: constellations and formations. The difference between the two methods lies in the relative positioning between satellites. Constellations are positioned relative to an object, such as the Earth, while in formations spacecraft are positioned relative to each other. Each satellite communicates with the others and shares the processing, communications, and payload or mission functions. Thus the cluster of satellites forms a "virtual satellite". This concept promises many benefits, including greater utility and flexibility by allowing the cluster to reconfigure and optimise its geometry for a given mission, enhanced survivability, and increased reliability.

In general terms, the formation-flying approach has the following advantages: the opportunity of completing space observation missions without large and expensive ground infrastructures, reducing operational costs. The deeper covering of the phenomena under observation, since different instruments, under different points of view, inspect it at the same time. By substituting one large complex satellite with a group of small satellites, a better flexibility is achieved, with the chance of reconfiguring the system in case of malfunction, thus avoiding the mission failure. The failure of one spacecraft will not compromise the mission. The employment of identical platforms within the constellation allows a standardisation of the manufacturing, thus reducing production costs. The system functionality is not extremely dependent from technology: it is possible to launch a temporary formation with state-of-the-art instruments and later increasing the system performance by adding one or more spacecraft to the formation. On the other hand, the development of formation-flying presents the following technological challenges: accurate sensors are needed to allow a precise determination of the state of the system in order to control the formation. High precision in spacecraft coordination is indispensable in order to avoid troubles linked with the reciprocal distances among the elements of the formation, most of all collisions between the satellites.

Current studies in spacecraft formation control vary from individual satellite control to the use of stochastic algorithms [Gurfil et al., 2002]. The main problem to be addressed in formation control is that of workload. For small, Earth centred formations, individual control is a viable options. As the satellite number and operational distance from Earth increase, methods that automate the control processes become a necessity. The method proposed here aims, to drastically reduce the workload required to control the formation. The potential function control method represents a means of both estimating the desired states of a spacecraft's location, and autonomously correct and control these states. It is based on Lyapunov's method for stability analysis and its efficiency in the problem of collision avoidance is due to the fact that it aims to avoid a particular condition rather than to reach a state of equilibrium.

2 Formation Flight Dynamics

We will now introduce the model used for formation dynamics, including the simplifications used. We assume that the Sun-Earth system is not disturbed by the inclusion of a third infinitively small body. The whole system rotates with a constant angular velocity ω, about the baricentre G, the position of which is a function of the Earth mass m_{E} and the Sun mass m_{S} as shown in Figure 2.1. The Earth is in a circular orbit around the Sun at distance R_{ES}. Additionally, Sun and Earth are supposed to be perfect spherical bodies.

The acceleration of a spacecraft M in the inertial frame R_0 is:

$$\mathbf{a}_{M/R_0} = \mathbf{a}_{M/R} + \mathbf{a}_{M\in R/R_0} + 2\mathbf{\Omega}_{R/R_0} \times \mathbf{v}_{M/R}\ ,$$

where $\mathbf{a}$ denotes an acceleration, $\mathbf{v}$ a velocity and Ω an angular velocity. In the local frame the acceleration becomes:

$$\mathbf{a}_{M/R_0} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} - \omega^2 \begin{bmatrix} R_{GL2} + x \\ 0 \\ z \end{bmatrix} - 2\omega \begin{bmatrix} \dot{z} \\ 0 \\ -\dot{x} \end{bmatrix}, \tag{1}$$

where R_{GL2} is the distance from G to L2.

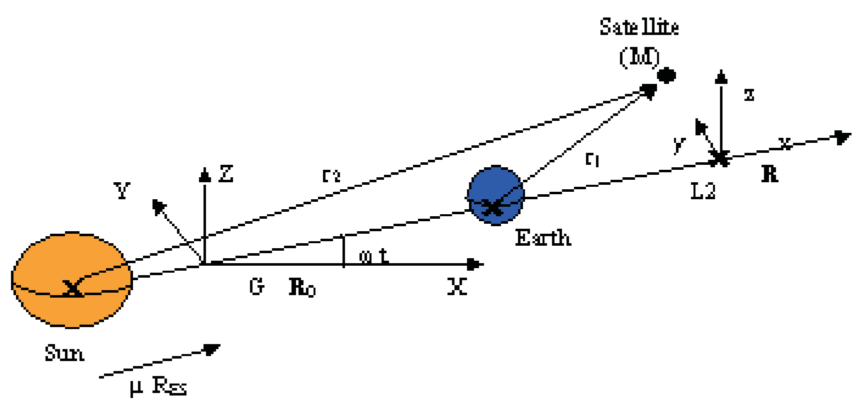

Figure 2.1: Geometric Configuration.

The forces f acting on the spacecraft are gravitational attraction and non-Newtonian. Because of the small size of satellites we confuse their centre mass with their gravitational centre. Therefore the equations of motion of a satellite are:

$$\mathbf{a}_{M/R_0} = -\mu_E \frac{\mathbf{r}_1}{\|\mathbf{r}_1\|^3} - \mu_S \frac{\mathbf{r}_2}{\|\mathbf{r}_2\|^3} + f,$$

where μ_{E} and μ_{S} are respectively the earth and the sun gravitational constants. In component form the equations of motion become:

$$\begin{aligned}
\ddot{x} - 2\omega\dot{z} - \omega^2(R_{\mathrm{GL2}} + x) + \frac{\mu_{\mathrm{E}}(R_{\mathrm{EL2}} + x)}{r_1^3} + \frac{\mu_{\mathrm{S}}(R_{\mathrm{SL2}} + x)}{r_2^3} &= f_{\mathrm{x}}, \\
\ddot{y} + \left(\frac{\mu_{\mathrm{E}}}{r_1^3} + \frac{\mu_{\mathrm{S}}}{r_2^3}\right) y &= f_{\mathrm{y}}, \\
\ddot{z} + 2\omega\dot{x} - \omega^2 z + \left(\frac{\mu_{\mathrm{E}}}{r_1^3} + \frac{\mu_{\mathrm{S}}}{r_2^3}\right) z &= f_{\mathrm{z}},
\end{aligned} \tag{2}$$

where R_{EL2} and R_{SL2} respectively the distances from the Earth and the Sun to the L2 point.

It is clear that these equations are non-linear as r_1 and r_2 hinge on the satellite position into the local frame. Moreover, the x and z variables are coupled whereas y is independent [Alfriend et al., 2002]. The majority of studies dealing with formation flying adopt linear equations of motion. The linearisation is very accurate since the distances of the satellites from the L2 point are very small compared to the distances between the L2 point and the Earth and the Sun [Hamilton et al., 2002]. The linearisation yields:

$$\begin{aligned}
\ddot{x} - 2\omega\dot{z} - (\omega^2 + 2\mu_0^2)x &= f_{\mathrm{x}}, \\
\ddot{y} + \mu_0^2 y &= f_{\mathrm{y}}, \\
\ddot{z} + 2\omega\dot{x} + (\mu_0^2 - \omega^2)z &= f_{\mathrm{z}},
\end{aligned} \tag{3}$$

with

$$\mu_0^2 = \frac{\mu_{\mathrm{S}}}{R_{\mathrm{SL2}}^3} + \frac{\mu_{\mathrm{E}}}{R_{\mathrm{EL2}}^3}.$$

Equation (3) will be used to model the relative motion of each satellite from the reference orbit.

3 The Potential Function Control Method

A dynamical system is stable in the sense that it returns to equilibrium after any perturbation, if and only if, there exists a Lyapunov function; some scalar function $V(x)$ of the state with the following properties.

Let $\dot{x} = f(x)$, $f(0) = 0$ and $0 \in \Omega \subset \Re^n$. If there exists a C^1 function $V: \Omega \to \Re$ such that:

(1) $V(0) = 0$;

(2) $V(x) > 0 \; \forall x \in \Omega, \; x \neq 0$;

(3) $\dot{V}(x) \leq 0 \; \forall x \in \Omega$,

than $x = 0$ is locally stable. Furthermore, if

(4) $\dot{V}(x) < 0 \; \forall x \in \Omega, \; x \neq 0$,

then $x = 0$ is locally asymptotically stable. This theorem can be easily modified by replacing (3) by $\dot{V}(x) \geq 0$ for limited times, which implies that $\dot{V}(x) \leq 0$ after a certain time and then the initial theorem is fulfilled.

The method then consists in controlling V to fulfil Condition 3 of Lyapunov's theorem. The time derivative of V is:

$$\dot{V} = \nabla f \cdot \mathbf{v},$$

where $\mathbf{v}$ is the velocity of the spacecraft. The velocity of the spacecraft will be controlled as:

$$\mathbf{v}_{\text{desired}} = -k \frac{\nabla V}{\|\nabla V\|},$$

where $\frac{\nabla V}{\|\nabla V\|}$ is the unit vector normal to the isopotential surface and k is a shaping parameter which regulates the amplitude of $\mathbf{v}_{\text{desired}}$, which is then analytic. When the control is switched on, the time derivative of the potential function is forced to be equal to:

$$\dot{V} = -\ k\,\|\nabla V\|,$$

which is non-positive thus fulfilling Lyapunov's theorem. Thus the cluster converges to the goal position avoiding collisions. Through the desired velocity, the path followed by the satellite is completely defined by the potential function. To optimise the time of the deployment and the fuel consumption this function must be shaped cleverly. Also, the velocity desired must be obtained efficiently despite of bounded thrusters and imperfect sensors.

3.1 Translational control

The component that controls the spacecraft translations will guide towards the goal positions while avoiding collisions. The attractive component, $V_{\text{a Trans}}$ is a function of the distance between the current spacecraft position and the desired final position while the repulsive component, $V_{\text{rep Trans}}$ is a function of the current spacecraft position and the obstacle position:

$$V_{\text{a Trans}} = \tfrac{1}{2}(\mathbf{r} - \mathbf{r}_f)^2 \quad V_{\text{rep Trans}} = A_{\text{T}}\ e^{-B_{\text{T}}(\mathbf{r}-\mathbf{r}_{\text{obs}})^2}. \tag{4}$$

With r the current spacecraft position, $\mathbf{r}_f$ the final position $\mathbf{r}_{\text{obs}}$ the obstacle position and A_{T} and B_{T} shaping parameters. One of the weak points of the potential function control method is the presence of saddle points in the potential function. The main problem consists in dimensioning the function around these points so that the satellites behave correctly there. We therefore develop a repulsive component that always maintains the same width. The parameter that controls the amplitude of the repulsive component now hinges on the distance from the target, $\mathbf{r}_{\text{target}}$, as well as the width of the obstacle d_{obs}:

$$A_{\text{T}} = \frac{1}{2}\ \frac{e^{B_{\text{T}} d_{\text{obs}}^2}}{B_{\text{T}} d_{\text{obs}}}\ \left(\mathbf{r}_{\text{target}} + \frac{1}{2 B_{\text{T}} d_{\text{obs}}}\right).$$

3.2 Rotational control

The component that controls the spacecraft rotation will guide towards the desired attitude while avoiding any constrained directions. The attractive component, $V_{\text{a Rot}}$ is a function of the angular distance between the current spacecraft attitude and the desired final attitude while the repulsive component, $V_{\text{rep Rot}}$ is a function of the current spacecraft attitude and the obstacle location:

$$V_{\text{a Rot}} = \ \tfrac{1}{2}\ \gamma^2 \quad V_{\text{rep Rot}} = \ A_{\text{R}}\ e^{-B_{\text{R}}\delta^2}\ . \tag{5}$$

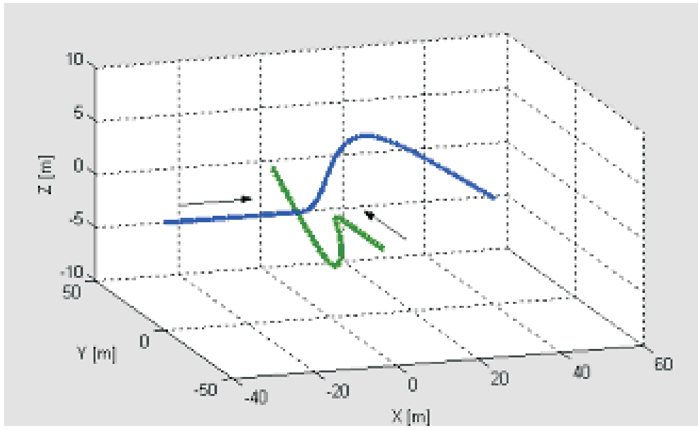

Figure 3.1: Translational Manoeuvre.

With A_{R} and B_{R} are shaping parameters and:

$$\gamma = \arccos(\mathbf{n}_{\mathrm{p}} \cdot \mathbf{n}_{\mathrm{f}}), \quad \delta = \arccos(\mathbf{n}_{\mathrm{p}} \cdot \mathbf{n}_{\mathrm{obs}}), \tag{6}$$

where $\mathbf{n}_{\mathrm{p}}$ is the unit vector along the payload axis, $\mathbf{n}_{\mathrm{f}}$ is the target unit vector and $\mathbf{n}_{\mathrm{obs}}$ is the unit vector along any avoidance directions. Once again we develop a repulsive component that always maintains the same width. The parameter that controls the amplitude of the repulsive component now hinges on the distance from the angular distance from the target, γ_{targ}, as well as the angular width of the obstacle δ_{obs}:

$$A_{\mathrm{R}} = \frac{1}{2}\,\frac{e^{B_{\mathrm{R}}\delta_{\mathrm{obs}}^2}}{B_{\mathrm{R}}\delta_{\mathrm{obs}}}\,\left(\gamma_{\mathrm{targ}} + \frac{1}{2B_{\mathrm{R}}\delta_{\mathrm{obs}}}\right).$$

In Figure 3.1 we see the avoidance action taken by two spacecraft on a colliding trajectory, while in Figure 3.2 we can see how the payload follows a trajectory from a random initial position to a desired target attitude, avoiding a constrained direction.

3.3 The potential function

We are now able to construct a Lyapunov function for each spacecraft that will guide them to their goal positions while avoiding collisions and avoiding restricted pointing directions. This function V consists of two components: attractive V_{a} and repulsive V_{rep}. Moreover, as we have seen the attractive and repulsive components will be made up of two parts each to account for the positional and attitude requirements. The potential function therefore will be:

$$V = \frac{1}{2}(\mathbf{r} - \mathbf{r}_f)^2 + \frac{1}{2}\,\gamma^2 + A_{\mathrm{T}}\;e^{-B_{\mathrm{T}}(\mathbf{r}-\mathbf{r}_{\mathrm{obs}})^2} + A_{\mathrm{R}}\;e^{-B_{\mathrm{R}}\delta^2}.$$

4 Plume Impingement Avoidance

Spacecraft thrusters send gas streams of various species onto spacecraft surfaces. The plume of gas particles emitted by thrusters may cause contamination, degradation or

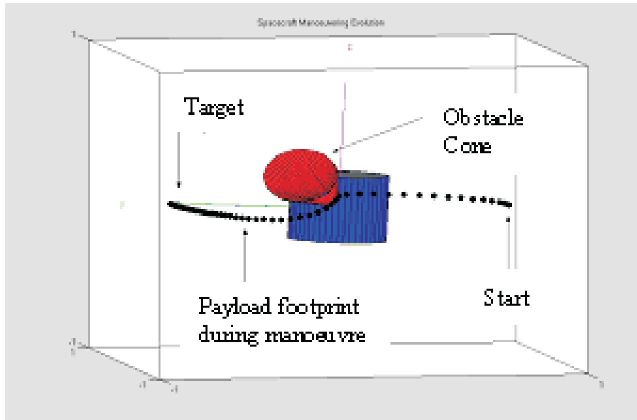

Figure 3.2: Rotational Manoeuvre.

damage to surface and can either directly or indirectly cause localized heating and contamination. Plumes and the resultant impingement phenomena are currently not well understood. Simple engineering models are used conservatively to estimate plume effects. The problem of plume impingement is a major concern for a cluster of spacecraft with close relative motion. The problem is compounded by the fact that when approaching each other, the spacecraft will have to fire the thrusters towards the incoming satellite to manoeuvre away from it. We intend to use the potential function method introduced earlier to address and solve this problem. The idea is to impart an attitude to the satellite that places the other spacecraft in a cone, where there are no exhaust particles, as shown in Figure 4.1.

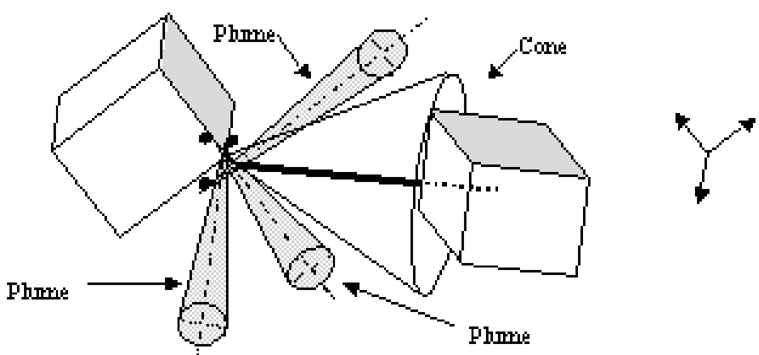

Figure 4.1: Plume Impingement Avoidance Cone.

To achieve this we make use of the potential method, and in particular the attitude control component explained in the previous section. We consider two unit vectors, $\mathbf{n}_t$ and $\mathbf{r}_s$, the first directed along the plume direction the second directed along the avoidance cone.

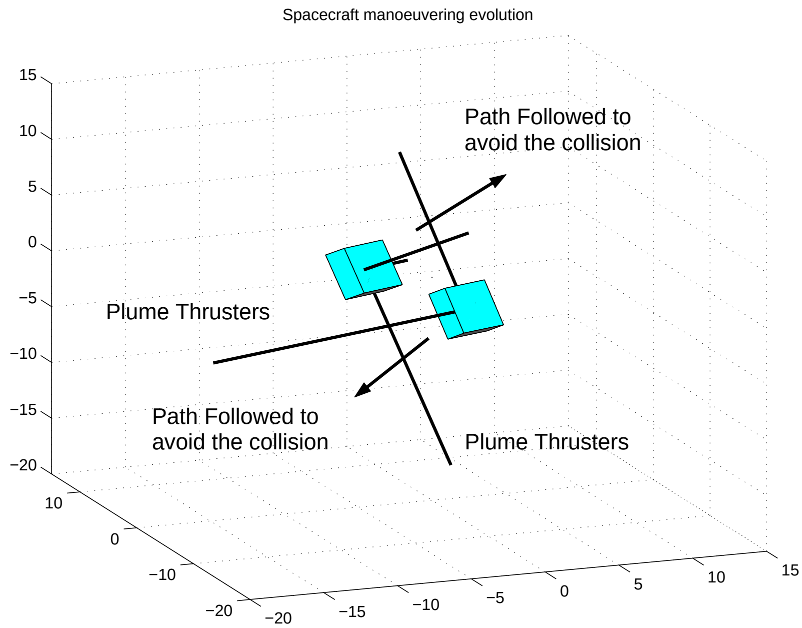

Figure 4.2: Plume impingement during deceleration phase.

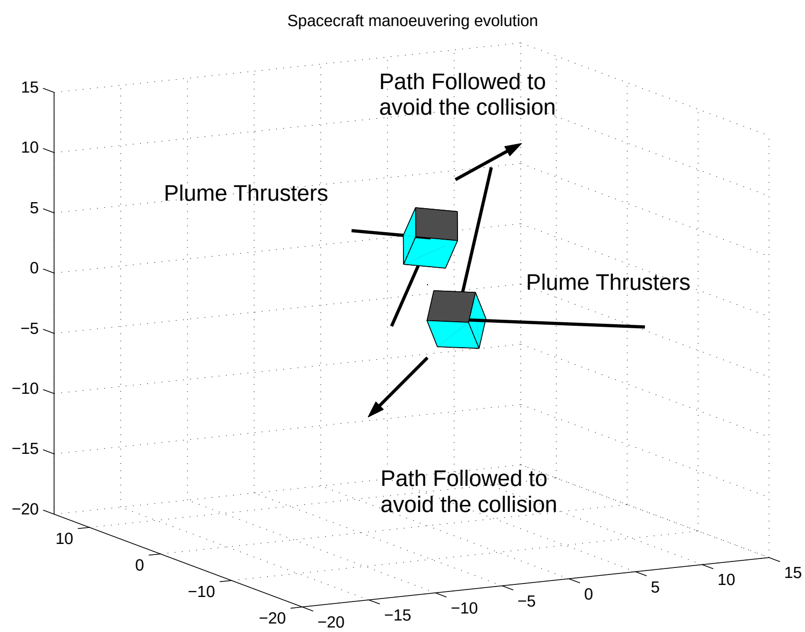

Figure 4.3: Plume impingement during avoidance phase.

The thrusters will then fire, if and only if, the plume lies outside the avoidance cone. In Figure 4.2 and 4.3 we can see that the method appears to be effective in avoiding plume impingement. At first two colliding spacecraft, decelerate their velocities, in Figure 4.2, before performing the avoidance manoeuvre, in Figure 4.3.

5 Conclusions

In this paper we have proposed some corrections to the traditional potential function control method. The strategies address the main drawback of such method, the presence of local minima. The methodology has been applied to a formation of spacecraft placed at the Sun-Earth L2 libration point. The control has been implemented for both translational and rotational movements, ensuring that the spacecraft reach the desired position with the required attitude. Obstacles, in the shape of other spacecraft or avoidance payload pointing areas have been accounted for in developing the potential function. Finally, the method has been used to avoid the possibility of plume impingement with promising results.

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